Protokoll: Winkelkorrelation

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Gruppe 101

Contents

Part A
Basics

1 Aim of this Measurement

The aim of the experiment is the measurement of gamma-gamma correlation of ^{60}Ni and the determination of the anisotropy and the correlation function.

Note: Since we did the theoretical basics of α-, β-, γ-decay, of interaction with matter and of scintillator counters already in other protocols in German, we won't quote it here again because we would have to translate everything into English and there wouldn't be any learning effect doing so.

2 Angular correlation of gamma rays

The gamma radiation of electromagnetic transitions from J_1 to J_2 has to be isotropic because of symmetry if:

1. All starting states[1] are uniformly occupied.
2. All possible transitions are measured at once.

If we look at some dipole transitions with the selection rule $\Delta m = -1, 0, 1$ we would except isotropic radiation. In fact, the probabilities are:

$$W_{\Delta m=\pm 1} \mathrm{d}\Omega = \frac{3}{16}\pi \left(1 + \cos^2\theta\right) \mathrm{d}\Omega$$
$$W_{\Delta m=0} \mathrm{d}\Omega = \frac{3}{8}\pi \sin^2\theta \mathrm{d}\Omega$$

So the whole propability is (if we assume uniform occupancy)

$$W_{\mathrm{Sum}} \mathrm{d}\Omega = \sum_i W_i \mathrm{d}\Omega = \frac{3}{4}\pi \mathrm{d}\Omega$$

which is perfectly isotropic. In the Experiment we can only measure the whole intensity. If every state is occupied N_i times we get:

$$I_{\mathrm{Sum}} = \sum_i N_i W_i \mathrm{d}\Omega = \left(N_+ W_+ + N_- W_- + N_0 W_0\right) \mathrm{d}\Omega$$

If we assume thermodynamic equilibration the occupation number N_i are given by the Boltzmann-distribution[2]:

$$N_i \propto \exp\left[-\frac{m_i}{J} \cdot \frac{\mu B}{kT}\right] \qquad \Rightarrow \qquad \frac{N_{i+1}}{N_i} \approx 1 \quad \text{for small } B \text{ and high } T$$

So we can assume uniform occupancy.

[1] there are $2 \cdot J_1 + 1$ states possible with different $m = -J_1, ..., J_1$
[2] at least at high temperature

We will get an anisotropy because we'll measure two transitions of one atom. We choose the quantisationaxis into the direction of the first photon. So $\theta = 0$. If we look at the dipole radiation as an example:

$$W_{\Delta m=\pm 1} \mathrm{d}\Omega = \frac{3}{16}\pi \left(1+\cos^2 0\right) \mathrm{d}\Omega = \frac{3}{8}\pi \mathrm{d}\Omega$$
$$W_{\Delta m=0} \mathrm{d}\Omega = \frac{3}{8}\pi \sin^2 0 \mathrm{d}\Omega = 0$$

So we know, that after the first transitions the states $m = 0$ isn't occupied and we'll get the anisotropy.

In general the differential cross section is given by:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \sum_{i=0}^{i_{\max}} A_{2i} \cdot P_{2i}(\cos\theta) \qquad i_{\max} = \min(L_1, L_2, J_2)$$

Here L_1, L_2 describe the initial and end state, while J_2 refers to the intermediate state. In the case of ^{60}Co we have quadrupole radiation. Hence,

$$L_1 = L_2 = J_2 = 2 \qquad \Rightarrow \qquad i_{\max} = 2$$
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = A_0 + A_2 \cdot P_2(\cos\theta) + A_4 \cdot P_4(\cos\theta)$$

Since we don't want a dependency on A_0 we define a korrelationfunction $K(\theta)$ by:

$$K(\theta) := \frac{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\theta)}{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(90^\circ)} = 1 + a_2\cos^2(\theta) + a_4\cos^4(\theta)$$

a_2 and a_4 are new constants which we want to measure. The theoretical values are:

$$a_2 = \frac{1}{8} \qquad a_4 = \frac{1}{24}$$

Last but not least, we define the anisotropy An by

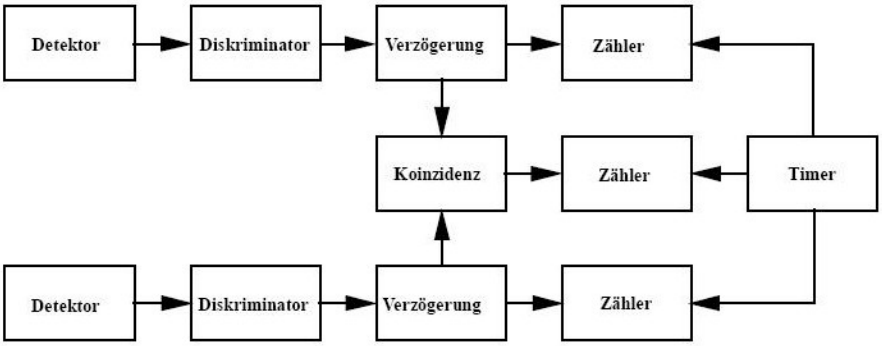
$$An := \frac{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(180^\circ) - \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(90^\circ)}{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(90^\circ)} = \frac{K(180^\circ) - K(90^\circ)}{K(90^\circ)}) = K(180^\circ) - 1 = a_2 + a_4$$

Part B
Experiment

In this Experiment we want to measure the gamma-gamma correlation of ^{60}Ni and determine the anisotropie. In order to do that we need to measure coincidences.

3 The Setup

The experiment is a typical correlation experiment setup. We use two NaJ-Scintillor-detectors. One of the detectors is fixed, whereas the other one can be positioned at the emission angles[3] of 90°, 135° and 180°. The basic principle of a coincidence measurement can be seen in the picture below.



In the setup a energy discriminator is used. In our experiment we set the lower level of the discriminator at 60-70% of the photopeak. This lower level is a compromise. On the one hand, we want to avoid detecting so called "false" coincidences, which occur due to compton scattering at the detectors. Those false coincidences have a energy lower than that of the photopeak, so by setting the lower level at the photopeak we eliminate those. But on the other hand we need to detect enough real coincidences in oder to have a good statistic.

After the discriminator a time delay can be set. We will use this delay to determine a rate of random coincidences. In addition to the rates measured by the two detectors we also obtain the rate of coincidences (e.g. how often both detectors detect a gamma-quant at the same time). These rates allow us to calculate the correlation function and the anisotropie.

Since we want to detect as many coincidences as possible it seems advantageous to use a sample with a high activity. However, this is not completely true, because one has also to take random coincidences into consideration. The number of true coincidences increases linearly with the activity, but the number of random coincidences is proportional to the square of the activity. In order to reduce the number of random coincidences, one has to choose a low resolutiontime τ_A. We will also determine τ_A in our setup.

$$\tau_A = \frac{N_Z}{N_1 \cdot N_2}$$

N_Z = random coincidences; $\quad N_1, N_2$ = counts of detector 1,2

[3] The fixed detector detects the γ-rays with the emission angle of 0°

4 Measurement

We take three series of measurement. In each series we measure the rates of the detectors and the coincidences for the three angles of 90°, 135° and 180° for 400s. In order to assure that the results are reproduceable, we measure each angle twice in one series.

Furthermore we measure the background radiation and the random coincidences.

Part C
Analysis

There are two different ways how to claculate the correlation function and the anisotropy.

- We sum up all the coincidence rates belonging to one angle and use those to determine the coefficients a_2 and a_4 in the correlation function. This way we can obtain the statistical error.
- We evaluate each series of measurement on its own and take the average later. This way we can obtain the total error.

5 Correlation function

As stated above the Correlation function is of the form:

$$K(\Theta) = 1 + a_2 \cos^2 \Theta + a_4 \cos^4 \Theta$$

We obtain K(Θ) by:

$$K(\Theta) = \frac{N_K(\Theta)}{N_1(\Theta)N_2(\Theta)} \frac{N_1(90°)N_2(90°)}{N_K(90°)}.$$

So we chose $K(90°) = 1$ to be normalized. In the following we will use the defintion:

$$K'(\Theta) = \frac{N_K(\Theta)}{N_1(\Theta)N_2(\Theta)} \qquad K(\Theta) = \frac{K'(\Theta)}{K'(90°)}$$

Hence we can use $K(135°)$ and $K(180°)$ to calculate a_2 and a_4.

$$a_2 = 4K(135°) - K(180°) - 3$$

$$a_4 = 2K(180°) - 4K(135°) + 2$$

The anisotropy is given by:

$$An = K(180°) - 1$$

6 Measurement

6.1 Background

We did several measurements which all lasted exactly 400 s. First (and also at the end) the measure the background without any radioactiv material. The results were:

	Detector 1 - N_1	Detector 2 - N_2	Coincendenzes - N_K
1st Background measurement	1869	1796	6
2nd Background measurement	1518	1520	1

Since the first rate of coincendenzes is really a bit high[4], we used only the second one. We subtracted the second measurement from each other measurement before calculation anything further.

[4] indeed it's even higher than the rate of random coincidences we measured with the ^{60}Co

6.2 Random coincidences

In order to measure the random coincidences we set the time delay between the two detectors at $2 \cdot 63\,\text{s} = 126\,\text{s}$. The result of the measurement was[5]:

	Detector 1 - N_1	Detector 2 - N_2	Coincendenzes - N_K
random coincidences	72565	73000	2

So the random value of the correlation function is:

$$K'_{\text{random}} = \frac{N_K}{N_1 \cdot N_2} = 0{,}378 \cdot 10^{-9}$$

We will subtract this value from every further value of the correlation function.

6.3 1st analysis method

In this analyisis method we added up all the counting rates for each angle and used those values to calculate the coefficients a_2 and a_4 and the anisotropie An. If we also substract the background, we obtain the following rates:

Θ	counter 1	counter 2	coincidences
180°	411947 ±690	425516 ±700	1995± 45
135°	408504± 687	428864 ±702	1853±44
90°	406219±686	406689 ±686	1698 ±42

The error as given above of the counting rates $N^{\text{sum}}_{\text{corr}}$ (sum of all values corrected with the background) can be calculated using Gaussian error propagation. Since this is a statistical measurement the error of one counting rate is given by its squareroot.

$$N^{\text{sum}}_{\text{corr}} = \sqrt{N^{\text{sum}} + 36 \cdot N^B}$$

N^{sum}=sum of all counting rates to one angle; N^B=background counting rate for this angle

With those counting rates we can calculate $K'(\Theta)$. The formula to calculate the error of $K'(\Theta)$ is:

$$\Delta K' = K' \cdot \sqrt{\left(\frac{\Delta N_K}{N_K}\right)^2 + \left(\frac{\Delta N_1}{N_1}\right)^2 + \left(\frac{\Delta N_2}{N_2}\right)^2}$$

Since we want to calculate $K(\Theta)$ we still need to correct K' by substracting K'_{random}. We obtain:

Θ	$K'_{\text{corr}}[10^{-9}]$
180°	11,02±0,44
135°	10,22±0,44
90°	9,92±0,44

$$\Delta K'_{\text{corr}} = \sqrt{(\Delta K')^2 + (\Delta K'_{\text{random}})^2}$$

mit $\Delta K'_{\text{random}} = 0{,}36$

Now we can calculate the correlation function for $\Theta = 135°$ and $\Theta = 90°$.

$$K(\Theta) = \frac{K'_{\text{corr}}(\Theta)}{K'_{\text{corr}}(90°)}; \qquad \Delta K(\Theta) = K(\Theta) \cdot \sqrt{\left(\frac{\Delta K'_{\text{corr}}(\Theta)}{K'_{\text{corr}}(\Theta)}\right)^2 + \left(\frac{\Delta K'_{\text{corr}}(90°)}{K'_{\text{corr}}(90°)}\right)^2}$$

[5]already corrected by the background

$$K(135°) = 1{,}030 \pm 0{,}064$$
$$K(180°) = 1{,}111 \pm 0{,}067$$

Thus we get for the coefficients:

$$a_2 = 4K(135°) - K(180°) - 3; \qquad \Delta a_2 = \sqrt{(4 \cdot \Delta K(135°))^2 + (\Delta K(180°))^2}$$
$$\Rightarrow \quad a_2 = 0{,}0093 \pm 0{,}2634$$

$$a_4 = -4K(135°) + 2(180°) + 2; \qquad \Delta a_4 = \sqrt{(4 \cdot \Delta K(135°))^2 + (2 \cdot \Delta K(180°))^2}$$
$$\Rightarrow \quad a_4 = 0{,}1019 \pm 0{,}2876$$

And for the anisostropie:

$$An = K(180°) - 1; \qquad \Delta An = \Delta K(180°)$$
$$\Rightarrow An = 0{,}111 \pm 0{,}067$$

The statistical error of the coefficients is extremly high. Therefore it is not surprising that our values differ greatly form the theoretical values. In order to determine if the method can nevertheless be used to calculate those if we only measure long enough and take enough series, we use a second analysis method which gives us the total error.

6.4 2nd analysis method

Here were added only the values of the two consecutive measurements of each angle. So there are three values for each angle. The resulted values for K' were:[6]

angle Θ	$K'(\Theta)[10^{-9}]$
180°	33,82
135°	33,54
90°	29,00
180°	32,52
135°	30,95
90°	30,59
180°	34,99
135°	29,55
90°	31,85

With these values we can calculate $K(\Theta) = \frac{K'(\Theta)}{K'(90°)}$. Note that $K(90°) = 1$ by definition.

	$K(180°)$	$K(135°)$	a_2	a_4	An
1st series	1,166	1,156	0,460	-0,293	0,166
2nd series	1,063	1,012	-0,015	0,079	0,063
3rd series	1,099	0,928	-0,387	0,486	0,099

[6]already corrected by the background and K'_{random}

In this table also the values for a_2, a_4 and An were already calculated by:

$$a_2 = 4K(135°) - K(180°) - 3$$

$$a_4 = 2K(180°) - 4K(135°) + 2$$

$$An = K(180°) - 1$$

We see, that the values of a_i and An are spreaded widely. Nethertheless we calulated the mean value and the derivation:

$$\bar{a}_2 = \frac{1}{3}\sum_i a_{2_i} = 0{,}019 \qquad \sigma_{a_2} = \sqrt{\frac{1}{3\cdot 2}\sum_i (\bar{a}_2 - a_{2_i})^2} = 0{,}245$$

$$\bar{a}_4 = \frac{1}{3}\sum_i a_{4_i} = 0{,}090 \qquad \sigma_{a_4} = \sqrt{\frac{1}{3\cdot 2}\sum_i (\bar{a}_4 - a_{4_i})^2} = 0{,}225$$

$$\bar{An} = \frac{1}{3}\sum_i An_i = 0{,}109 \qquad \sigma_{An} = \sqrt{\frac{1}{3\cdot 2}\sum_i (\bar{An} - An_i)^2} = 0{,}030$$

The values of a_i don't fit the theoretical values $a_2 = 0{,}125$ and $a_4 = 0{,}042$ very well. But the errors are that high that the theoretical values are still in the error boundaries.

6.5 Conclusion

In the both methods we calculated the following two sets of results. Except for the anisotropy in the 2nd method, the theoretical values are always between the error boundaries. Since there's only a chance of 68,3% for beeing in the standard deviation, we don't need to worry about that.

	1st method	2nd method	theory
a_2	$0{,}009 \pm 0{,}263$	$0{,}019 \pm 0{,}245$	0,125
a_4	$0{,}102 \pm 0{,}288$	$0{,}090 \pm 0{,}225$	0,042
An	$0{,}111 \pm 0{,}067$	$0{,}109 \pm 0{,}030$	0,162

The errors of the 1st method are pure statistical errors. They could be minimized if one would do a longer measurement. The errors of the 2nd method are somehow combined errors of the statistical and the systematical errors. If they were significally larger then the 1st one, there wouldn't be any benefit in doing a longer mesurement.

Since the statistical errors are slightly larger, one could do a longer measurement unless the statistical errors will get much smaller than the error out of 2nd method.

6.6 resolution time τ_A

Moreover we have to calculate the resolution time. It is given by:

$$\tau_A = \frac{N_Z}{N_1 N_2}\cdot T; \qquad \frac{\Delta\tau_A}{\tau_A} = \sqrt{\left(\frac{\Delta N_Z}{N_Z}\right)^2 + \left(\frac{\Delta N_1}{N_1}\right)^2 + \left(\frac{\Delta N_2}{N_2}\right)^2}$$

T is the measurement period $T = 400$s. The rates N_Z, N_1 and N_2 were of course corrected with the background. Thus we get:

$$\tau_A = (210 \pm 70)\text{ns}$$

We know that the experiment is supposed to have a minimum resolution time of 10ns. So our result seems plausible.